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A Note on Optimization methodology for Non-linear Function under Heuristic Approach

Abstract

In the optimization problems, there are good numbers of theories to justify the optimality of a function for both linear and nonlinear. However, in practical approach, establishing convexity and concavity for a non-linear function with more than three variables is a herculean task and in those cases we usually consider the heuristic approach. Although the optimum of a function may not exist in the assumed search boundaries and we usually stuck with maximum or minimum value of the function. In this note we propose a methodology to address optimization problem with more than three variables using heuristic approach.

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1. Introduction

In the field of computational mathematics, optimization is a active area in both pure and applied mathematics. The application of optimization can be observed in the area of inventory control (Bhattacharjee & Sen, 2021a, 2021b), lot sizing (Wu et al. 2012), scheduling (Tamara etc al. 2017) etc.The optimization of functions with single one and two variable function is attainable through analytical approach. However, for a non-linear function with large number of variables and implicit expression of partial derivatives, the analytical approach seems to be a bit tedious and sometime impossible to approach. In such cases various solution procedures and methodologies are proposed to obtain a suitable optimal solution for the function. Some of the solution procedures follow heuristic approaches where the function is optimized through two or more different algorithms and a comprehensive analysis is made to justify the optimality of the solution. Although there are certain situations where we stuck with maximum or minimum value of the function only, they need not be optimal; such situation arises when there is no critical point in the search boundary.

In this note, an effort has been given to propose an optimization methodology for higher dimensional nonlinear function using heuristic approach.

2. Methodology

Let $D \subset \boldsymbol{R}^{\boldsymbol{n}}$be a search domain of the function f defined on D. Let $f be a C^2 -$ function of n variables $x = (x_1, x_2, \dots, x_n)$. In order to obtain an optimal value of $f$the following steps are proposed,

1. We define the function, $G(x) = \sum_{1 \leq k \leq n} |f_{xk}(x)|$ for all $x \in D$
2. It is clear that function G is the finite sum of positive real number therefore, min$G = 0$
3. Obtain the value of $x = \alpha$for which $G$is minimum and consequently $f_{xk}(\alpha) = 0$for all $1 \leq k \leq n$. Therefore $x = \alpha$ is a critical point of the function f
4. Let $B \subseteq D$ be the new search domain cantered at $x = \alpha$
5. Define the function $h(x) = f(x) - f(\alpha)$ for all $x \in B$
6. Now there are three possibilities
    - If min $h > 0$then the function f has a global minimum at $x = \alpha$
    - If max $h < 0$ then the function f has a global maximum at $x = \alpha$
    - If neither of the above exists then the function has neither maximum nor minimum value

Note 1: In the above stated methodology Step 3 and Step 6 can be performed using any heuristic technique and algorithms. Thus, the optimization of $f$can be established using heuristic approach and Statistical verification in not mandatory for this approach as we are not finding whether the sequence of global best converges or not, rather we are applying the definition of maximum and minimum to justify the optimization.

Note 2: Step 1-3 can be computed manually if it is possible to express the variables explicitly and in that case one can directly begin from Step 4. Otherwise, for implicit expression of partial derivatives f_{xk} one can apply the method from Step 1.

Note 3: It is possible that the critical point obtained using Step 3 is not suitable for the study purpose in that case one can change either the search domain D or input the fresh parameters of the function whichever is applicable for the problem.

3. Conclusion

This is a note to provide a methodology to solve an optimization problem through heuristic approach. So, far the author's best knowledge this methodology of heuristic approach is not proposed to solve optimization problems. In this approach two functions are defined using the given function. First function is defined to obtain critical points, however, if the variables can be expressed explicitly in terms of parameters and other variables from the expression of partial derivatives then one can directly starts from Step 4. The proposed methodology is a soft computing approach to optimization problems with large number of variables. The present note can be extended to analyse some optimization problem.

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